

Improving probabilistic prediction of daily streamflow by identifying Pareto optimal approaches for modelling heteroscedastic residual errors

David McInerney, Mark Thyer, Dmitri Kavetski

Intelligent Water Decisions Research Group (www.waterdecisions.org)

University of Adelaide

George Kuczera

University of Newcastle

Julien Lerat

Seasonal Streamflow Forecasting, Bureau of Meteorology



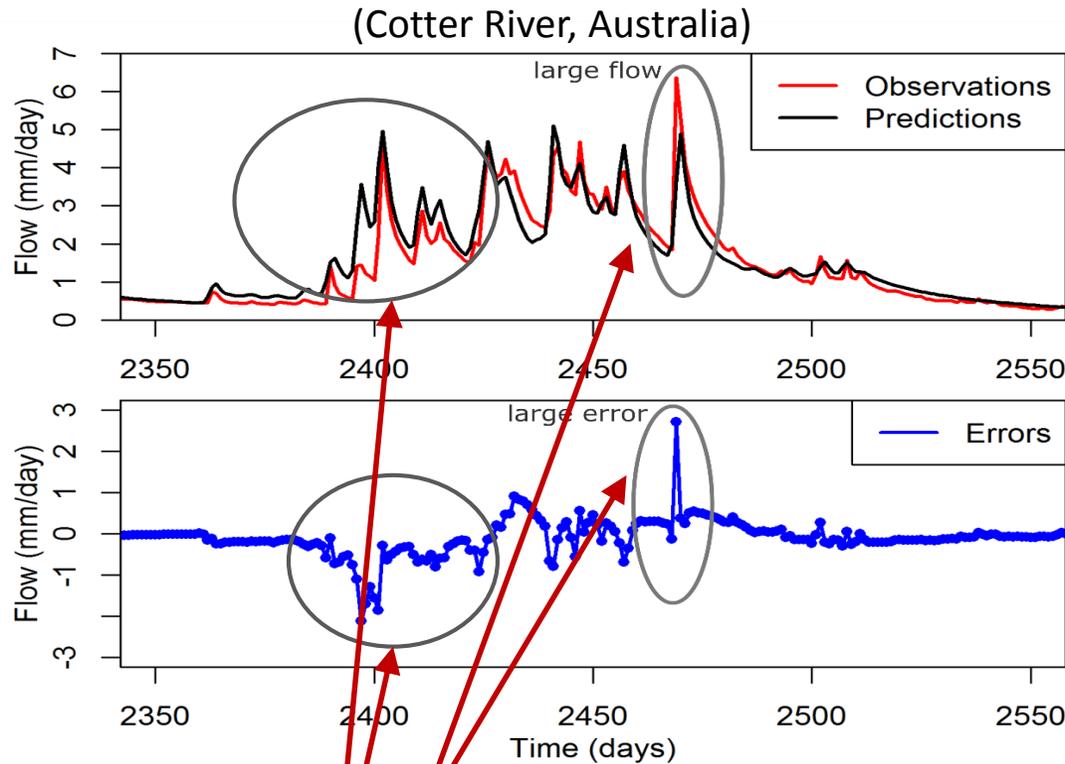
Motivation

- Evaluating uncertainty in hydrological predictions is important for decision making and risk assessment

Aims

- Improve probabilistic predictions of daily streamflow
- Comprehensive evaluation of residual error modelling approaches for representing predictive uncertainty
- Provide recommendations for different types of catchments

Challenging features of residuals in hydrology



**Streamflow
time series**

**Residual errors
time series**

Residual = observations -
predictions

- **Errors are heteroscedastic** (larger errors in large flows)
- **Errors have persistence** (not independent between time steps)
- **Key Challenge:** Identifying residual error models that represent both “features” to achieve reliable and precise probabilistic predictions

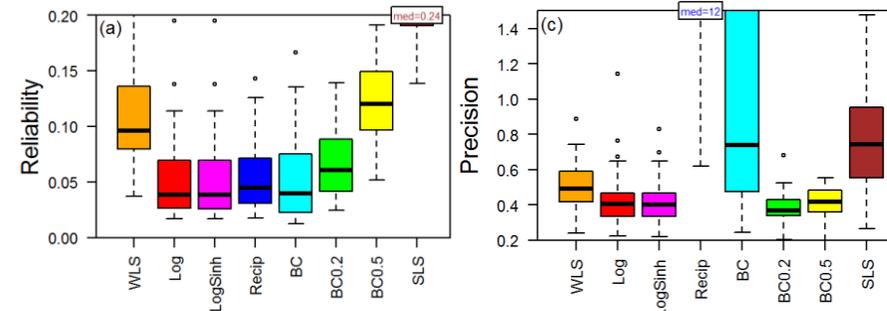
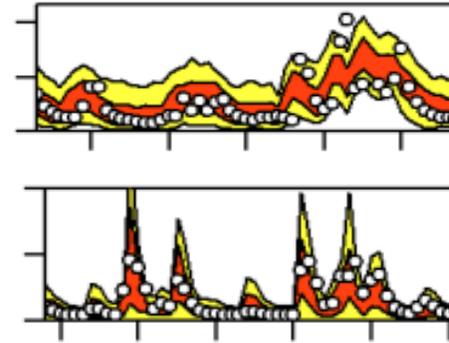
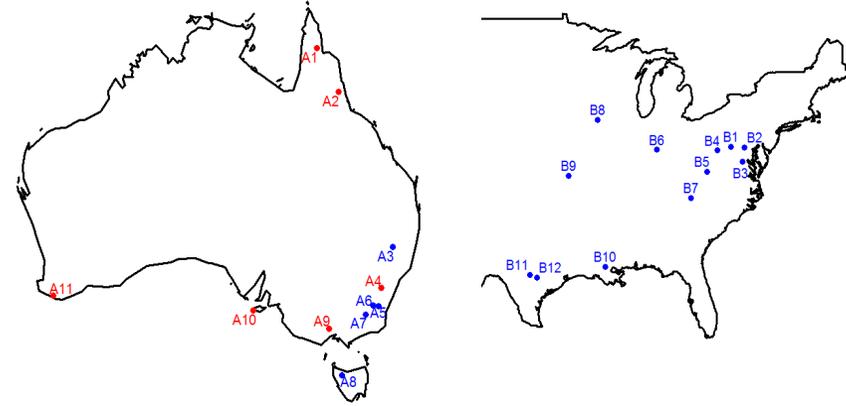
What is the “best” residual error model for making daily streamflow probabilistic predictions?

- Research Gap: No study had comprehensively compared the range of residual error models for representing heteroscedasticity in residuals

Residual Error Model	Description
No heteroscedasticity	
SLS	Standard least squares (error sd is constant)
Direct approaches for heteroscedasticity	
WLS	Weighted least squares (error sd increases linearly with predictions)
Transformational approaches for heteroscedasticity	
Log	Log transformation
Logsinh	Logsinh transformation (error sd increase “tapers off” with predictions)
BC (inferred λ)	Box-Cox transformation with inferred λ parameter
BC0.2	Box-Cox transformation with fixed $\lambda= 0.2$
BC0.5	Box-Cox transformation with fixed $\lambda= 0.5$
Reciprocal	Reciprocal transformation

Features of Comprehensive Evaluation

- Multiple catchments
- Two hydrological models
 - GR4J & HBV
- Multiple performance metrics
 - Reliability, precision, bias
- Theoretical insights to understand differences in performance
- Provide recommendations for perennial/ephemeral catchments
- Comprehensive evaluation improves robustness of recommendations



Key findings from empirical results

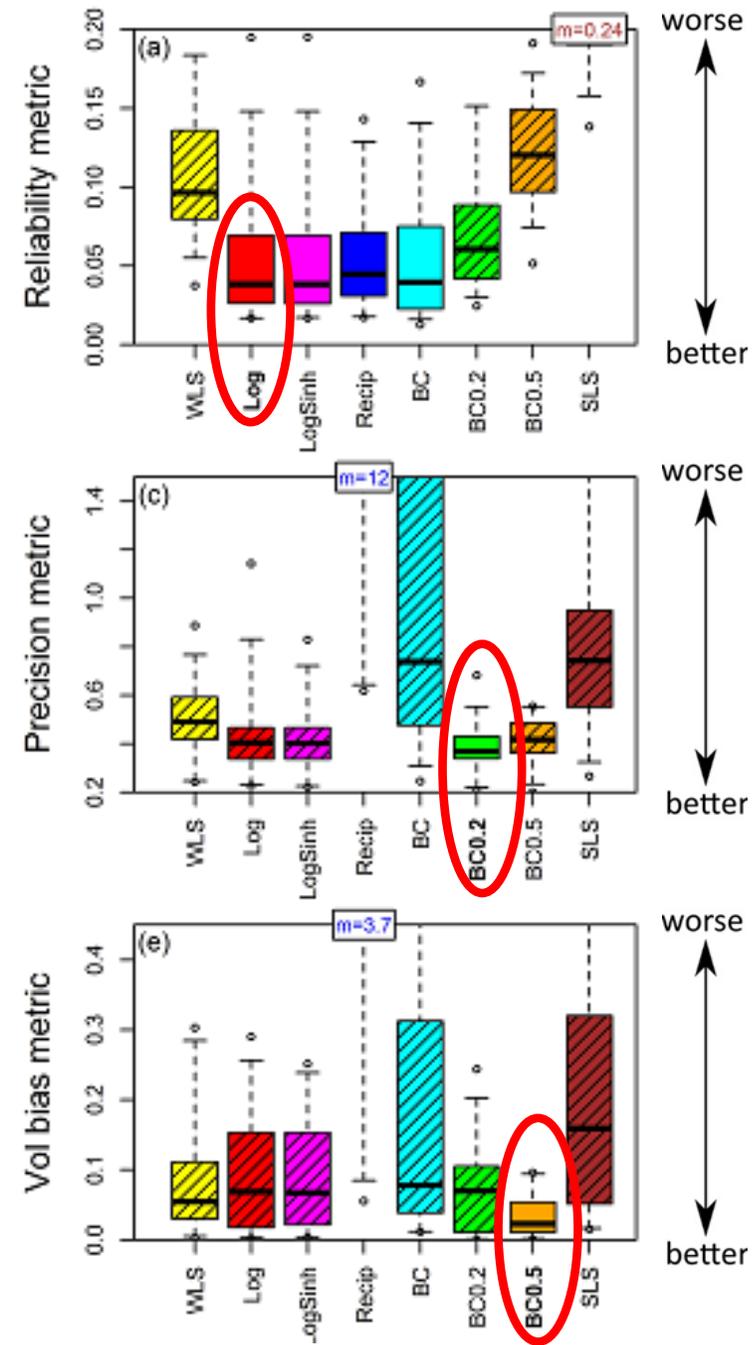
Perennial catchments

“Best” Residual Error Models

Model	Outcome
Log	Best reliability Good precision and bias
BC0.2	Best precision Good reliability and bias
BC0.5	Best bias

Not Recommended:

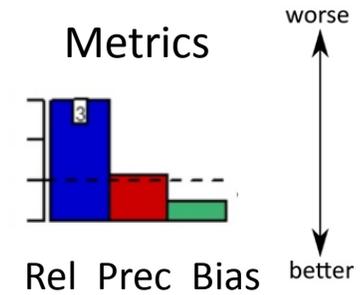
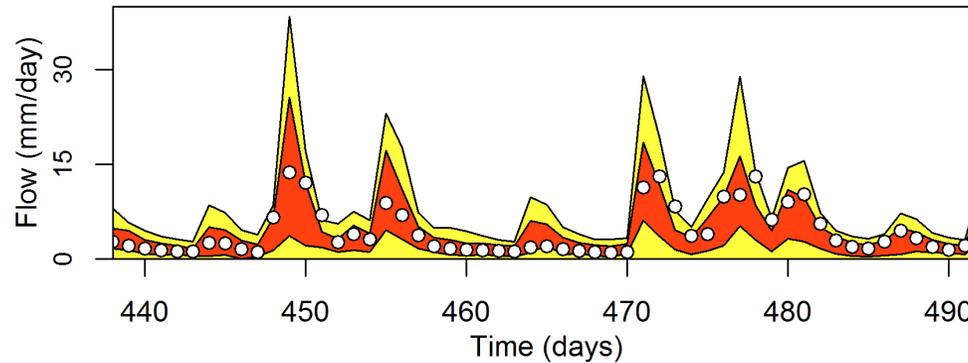
- Logsinh similar to Log, but more complex
- BC (inferred λ) poor precision
- Others have worse reliability, precision and bias



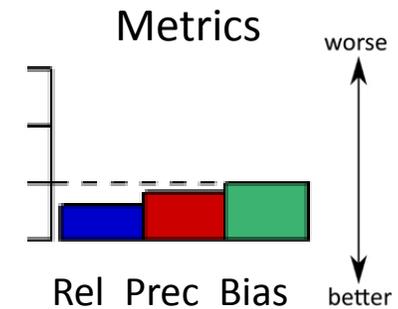
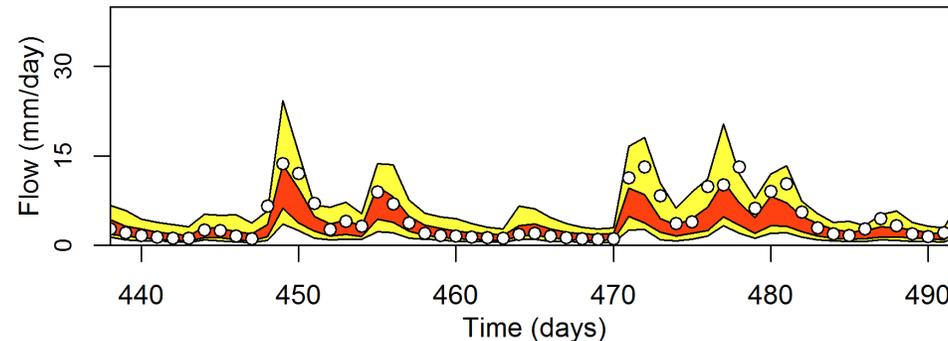
Transformational approaches (Log, BC) outperform direct approaches (WLS)

- Perennial catchment (Spring River, USA), GR4J hydro model

Weighted Least Squares (WLS)



Log transformation



- Log transformation better reliability and precision than WLS
- Theoretical Insight: Transformational approaches (Log and BC) better capture skew and kurtosis in observed residuals than WLS

Key findings from empirical results

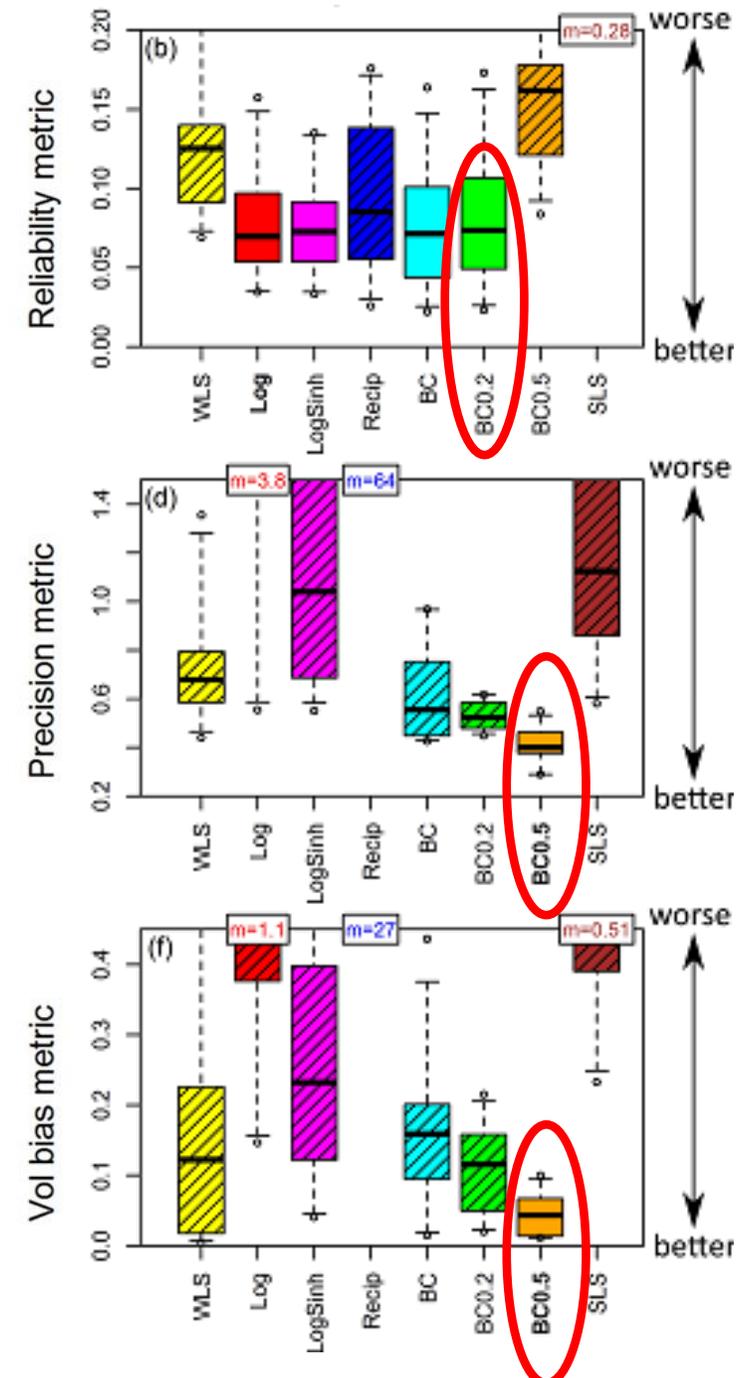
Ephemeral catchments

“Best” Residual Error Models

Model	Outcome
BC0.2	Equal best reliability Good precision and bias
BC0.5	Best precision and bias

Not Recommended:

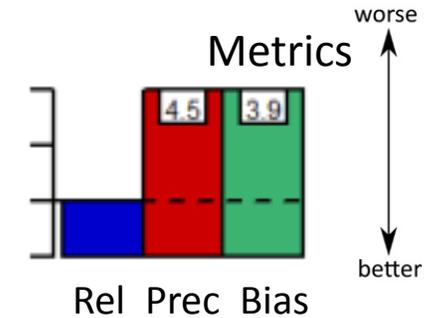
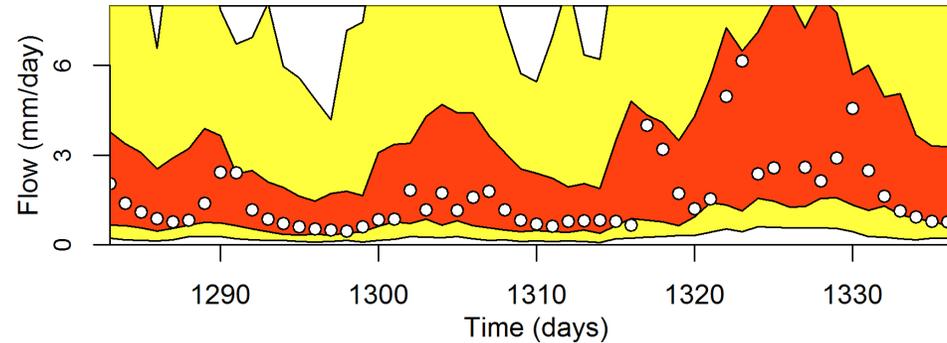
- Log/Logsinh poor precision
- BC (inferred λ) similar to BC0.2, but more complex
- Others worse reliability, precision and bias



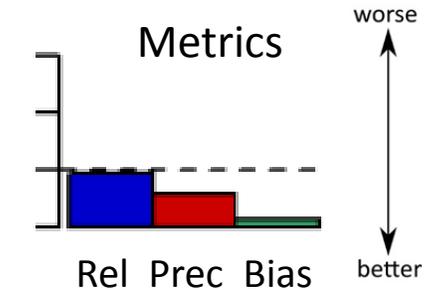
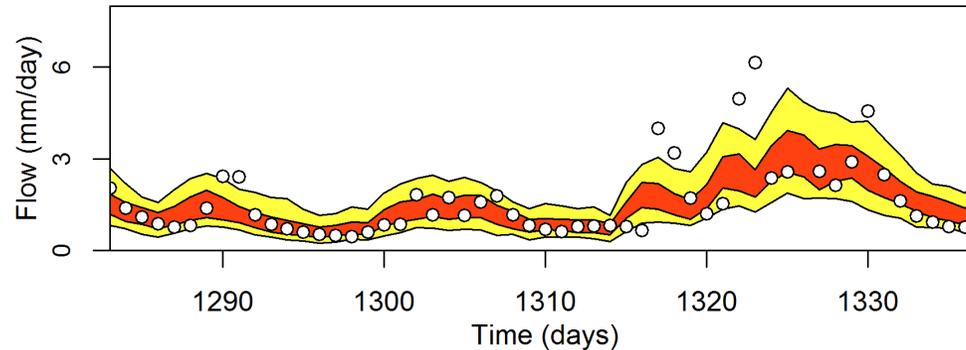
Box-Cox Transformation (fixed $\lambda=0.2$) outperforms log transformation in Ephemeral Catchments

- Ephemeral catchment (Rocky River, SA), HBV hydro model

Log transformation



Box Cox transformation ($\lambda=0.2$)

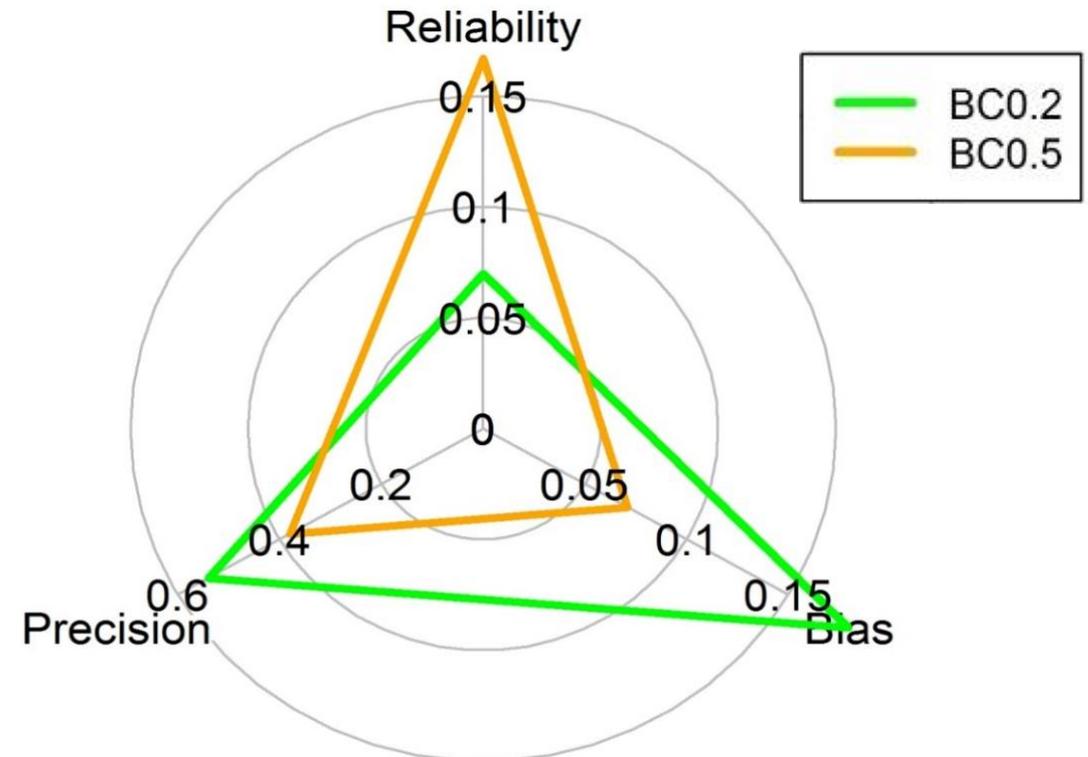


- BC0.2 has similar reliability, but much better precision and bias than log
- Log produces poor precision (unrealistically large uncertainty) and large bias in ephemeral catchments
- Theoretical Insight: BC transformation better handles zero flows than log

Using multiple performance metrics enables identification of “Pareto Optimal” error models

- Pareto Optimal Approaches
 - One performance metric can't be improved without worsening another metric
 - Perennial: Log, BC0.2 and BC0.5
 - Ephemeral: BC0.2 and BC0.5
- Allows user to select residual error model based on relevant metrics for practical application

Ephemeral Catchments



Broad Recommendations

In **perennial** catchments, use

- Log transformation if reliability is important
- Box Cox transformation with fixed $\lambda=0.2$ if precision is important
- Box Cox transformation with fixed $\lambda=0.5$ if low bias is important

In **ephemeral** catchments, use

- Box Cox transformation with fixed $\lambda=0.2$ if reliability is important
- Box Cox transformation with fixed $\lambda=0.5$ if precision/bias important

... but individual catchment results may differ from broad recommendations

Summary

- Comprehensive evaluation of approaches for predictive uncertainty
 - Multiple residual error models/catchments/hydro models/performance metrics
- Broad recommendations
 - Significant reductions in predictive uncertainty, while maintaining reliability
- Practical implications: Simplest is often best!
 - Smart use of simple approaches => best predictive performance
- Impacts
 - Operationalized in BOM dynamic seasonal forecasts [[Thyer, 2:25pm](#)]
 - DEWNR streamflow forecasting in SE South Australia [[Gibbs, 2:45pm](#)]

Water Resources Research

RESEARCH ARTICLE

10.1002/2016WR019168

Key Points:

- Choice of heteroscedastic error modeling approach significantly impacts on predictive reliability,

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David McInerney¹ , Mark Thyer¹ , Dmitri Kavetski^{1,2} , Julien Lerat³, and George Kuczera²